On Digital Filter Design with Semidefinite Programming

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Semidefinite matrices

Definition

Matrix $A \in \mathbb{R}^{n \times n}$ is *positive semidefinite*, denoted as $A \succcurlyeq 0$, if it is symmetrical and for each $x \in \mathbb{R}^n$: $x^T Ax \geqslant 0$.

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Positive semidefinite cone

Set of positive semidefinite matrices (of size $n \times n$) $S^n_+ = \{X \in S^n : X \succcurlyeq 0\}$ is a cone, called *the cone of positive semidefinite matrices.*

Semidefinite programming features

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Pros

- all LP problems can be formulated as SDP problems,
- many real-life problems in operations research can be modeled by SDP,
- easily solvable by interior-point methods (special case of conic programming).

Semidefinite programming — general form

Let us define

$$F(x) := F_0 + \sum_{i=1}^{m} x_i F_i,$$

where $F_0, F_1, \ldots, F_m \in S^n$ are symmetric matrices.

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Consider the following:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{st} \quad & F(x) \succcurlyeq 0. \end{aligned} \qquad (*)$$

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Proposition

A symmetrical block-diagonal matrix is positive semi-definite if and only if all the blocks are positive semi-definite.

Convexity

Fact

The problem defined above is convex.

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Justification

$$\forall x, y \in \mathbb{R}^m : \forall \lambda, \mu \in [0,1], \lambda + \mu = 1 \Rightarrow$$
$$\Rightarrow F(\lambda x + \mu y) = \lambda F(x) + \mu F(y) \succcurlyeq 0.$$

Since $x \mapsto F(x)$ is affine, and $c^T x$ is linear, then it is also a conic linear program.

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Namely — in digital signal processing (DSP).

What are digital filters?

Digital filters

Digital filters with discrete-time are systems processing the input signal $u : \mathbb{Z}_+ \to \mathbb{R}$ to output signal $y : \mathbb{Z}_+ \to \mathbb{R}$.

This is a *very* vague characterization.

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In general...

- A filter can be a Finite Impulse Response (FIR) filter, or
- an Infinite Impulse Response (IIR) filter.

How to describe digital filters?

Impulse response

Impulse response (IR) is a sequence $h : \mathbb{Z}_+ \rightarrow \mathbb{R}$

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FIR filters can be efficiently described by their *impulse response* — reponse y of the system to a Kronecker δ input (namely $[1\ 0\ 0\ \ldots]$).

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In case of IIR filters IR is less useful, since it may contain infinitely many non-zeros.

Definitions (FIR)

For an arbitrary input signal u:

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We will assume now that the IR h — is an absolutely convergent sequence. It means that the filter is stable (bounded input — bounded output).

Definitions (FIR)

For such systems the series

$$H(z) = \sum_{k=0}^{\infty} h_k z^{-k}$$

is absolutely convergent (for complex $|z| \geqslant 1$).
Such a function $H(z)$ is called *the transfer function*.

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Let $\omega \in [0, 2\pi]$ and consider the system's response to a (complex) input signal $u(t) = e^{j\omega t}$, for $t \in \mathbb{Z}$. It is easy to see that

$$y(t) = \sum_{\tau=-\infty}^{t} h_{t-\tau} e^{j\omega\tau} = \sum_{k=0}^{\infty} h_k e^{j\omega(t-k)} = e^{j\omega t} \sum_{k=0}^{\infty} h_k e^{-j\omega k}.$$

It means that $y(t) = H\left(e^{j\omega}\right) e^{j\omega t}$. And the function assigning ω a complex number $H\left(e^{j\omega}\right)$ is called **the frequency response**.

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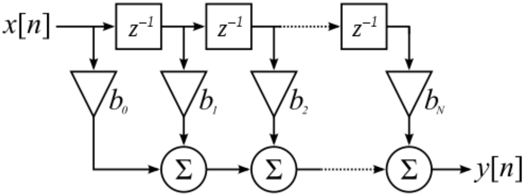
The smallest such n is called *the filter length*.
The transfer function of such filter is given by $H(z) = \sum_{k=0}^{n-1} h_k z^{-k}$.



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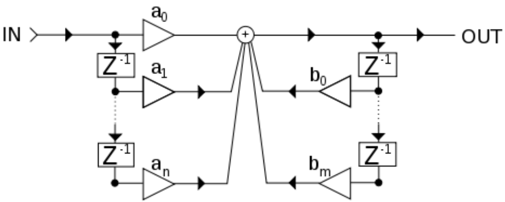
Pair (n, m) is called *filter length*.

IIR filter schematics



FIR/IIR filters comparison

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Infinite Impulse Response (IIR) Filters:

- an effort has to be made to make them stable,
- provide beter filter parameters for a given filter length,
- there are some valuable results thanks to SOCP methods.

Classical FIR Filter design methods:

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Linear Programming method (Rabiner)

- slower than Parks-McClennan (big problems),
- allows for some extra constraints during design.

Assumptions

System characteristics

We will now only consider FIR filters. Those systems are:

- linear,
- time-invariant,
- causal.

Let us consider FIR filter's frequency response:

$$H(e^{j\omega}) = \sum_{k=0}^{n-1} h_k e^{-jk\omega}.$$

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Denoting

$$c_k(\omega) = \cos(k\omega), \quad s_k(\omega) = \sin(k\omega)$$

we reformulate as

$$H(e^{j\omega}) = \sum_{k=0}^{n-1} h_k (c_k(\omega) - j s_k(\omega)).$$

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This problem can be easily reformulated as

$$\begin{aligned} \text{minimize} \quad & t \\ \text{st} \quad & |H(e^{j\omega}) - H^{\text{aim}}(\omega)|^2 \leqslant t, \omega \in \Omega^{\cdot} \end{aligned}$$

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Both above problems have infinite number of constraints.

We can also add weight $w : [0, 2\pi] \to [0, \infty)$ (since different importance for different ω).

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Discretization of ω!

- $H_i^{\text{aim}} = H^{\text{aim}}(\omega_i)$ discretized desired response for ω_i,
- $N_1 - 1$ frequencies ω_i in passband,
- $N - N_1 + 1$ frequencies in stopband.

After discretization the optimization problem becomes

$$\begin{aligned} \text{minimize} \quad & t \\ \text{st} \quad & w^2(\omega_i)|H(e^{j\omega_i}) - H_i^{\text{aim}}|^2 \leqslant t, i = 1, 2, \dots, N. \end{aligned}$$

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And denote as H_i^{real} — real part of H_i^{aim} and H_i^{im} — imaginary part.

Let us recall that

$$H(e^{j\omega}) = \sum_{k=0}^{n-1} h_k(c_k(\omega) - js_k(\omega)),$$

so

$$w^2(\omega_i)|H(e^{j\omega_i}) - H_i^{\text{aim}}|^2 =$$

$$= w^2(\omega_i)\left|\sum_{k=0}^{n-1} h_k(c_k(\omega_i) - js_k(\omega_i)) - H_i^{\text{real}} - jH_i^{\text{im}}\right|^2 =$$

$$= w^2(\omega_i)\left(\left[\sum_{k=0}^{n-1} h_k c_k(\omega_i) - H_i^{\text{real}}\right]^2 + \left[\sum_{k=0}^{n-1} h_k s_k(\omega_i) + H_i^{\text{im}}\right]^2\right).$$

And let us denote:

$$\alpha_1(\omega_i) := w(\omega_i)\Big[\sum_{k=0}^{n-1} h_k c_k(\omega_i) - H_i^{\text{real}}\Big],$$

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We can now reformulate the constraints as $\alpha_1^2(\omega_i) + \alpha_2^2(\omega_i) \leqslant t$. Observe that α_1 and α_2 depend on the vector of decision variables $(h_0, h_1, \dots, h_{n-1})^T$.

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Observe that α_1 and α_2 depend on the vector of decision variables $(h_0, h_1, \ldots, h_{n-1})^T$.
Let us proceed to the semidefinite formulation.

It has been shown in [3] that the constraints $\alpha_1^2(\omega_i) + \alpha_2^2(\omega_i) \leqslant t$ can be reformulated in terms of SDP.
Let us define (for each ω_i) a matrix

$$D_i := \begin{bmatrix} t & \alpha_1(\omega_i) & \alpha_2(\omega_i) \\ \alpha_1(\omega_i) & 1 & 0 \\ \alpha_2(\omega_i) & 0 & 1 \end{bmatrix}.$$

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It is positive semidefinite if and only if

$$\begin{aligned} t - \alpha_1^2(\omega_i) - \alpha_2^2(\omega_i) &\geqslant 0, \\ t - \alpha_1^2(\omega_i) &\geqslant 0, \\ t - \alpha_2^2(\omega_i) &\geqslant 0, \\ t &\geqslant 0. \end{aligned}$$

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It means that the each constraint $\alpha_1^2(\omega_i) + \alpha_2^2(\omega_i) \leqslant t$ is equivalent to $D_i \succcurlyeq 0$.
The decision variable is a vector $x = \big(t, h_0, h_1, \ldots, h_{n-1}\big)^T \in \mathbb{R}^{n+1}$.
Remembering that all D_i depend on x, we can finally write that

$$F(x) := \begin{bmatrix} D_1 & & & 0 \\ & D_2 & & \\ & & \ddots & \\ 0 & & & D_N \end{bmatrix}.$$

It is obvious that $F(x)$ is positive semidefinite if and only if each block D_i is positive semidefinite. It is also easy to see that $F(x)$ is an affine function of the decision variable x.

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Final SDP problem

Now we can reformulate the optimization problem as

$$\begin{aligned} \text{minimize} \quad & t \\ \text{st} \quad & F(x) \succcurlyeq 0. \end{aligned}$$

Where $x = \left(t, h_0, h_1, \ldots, h_{n-1}\right)^T$ is a vector of decision variables and the parameters (frequencies ω_i and desired frequency responces H_i^{aim}) are *hidden* in D_i.

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It is a semidefinite programming problem.

Numerical experiments

Consider a filter.

Filter's frequency response is given by

$$H^{\text{aim}}(\omega) = \begin{cases} e^{-j42\omega} & \text{for } 0 \leqslant \omega \leqslant 0.45\pi, \\ 0 & \text{for } 0.5\pi \leqslant \omega \leqslant \pi. \end{cases}$$

With an arbitrary filter length of $n = 85$.

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We will create 300 sample frequencies uniformly spread on the passband and stopband ($\omega_1, \ldots, \omega_{142}$ and $\omega_{143}, \ldots, \omega_{300}$).

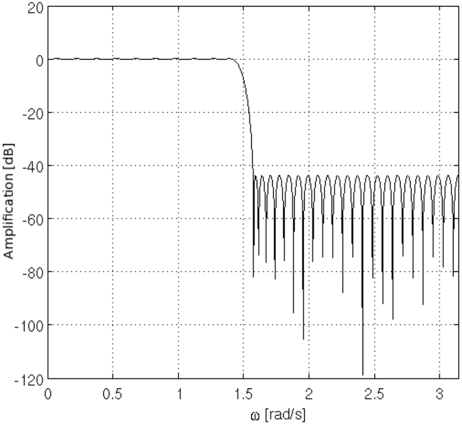
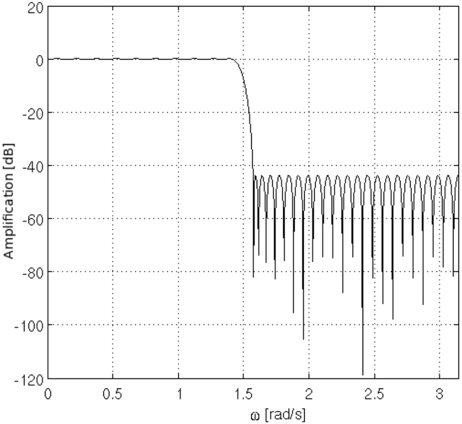


Figure: Absolute value of the transfer function $H(e^{j\omega})$ of the designed low-pass filter



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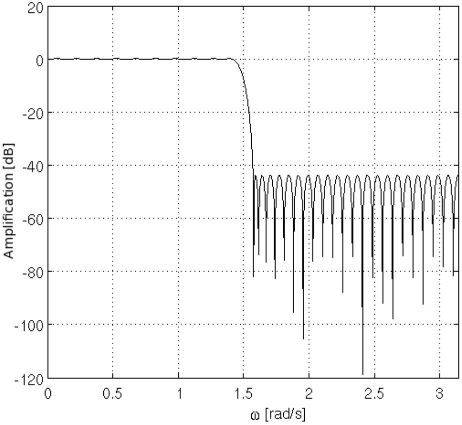

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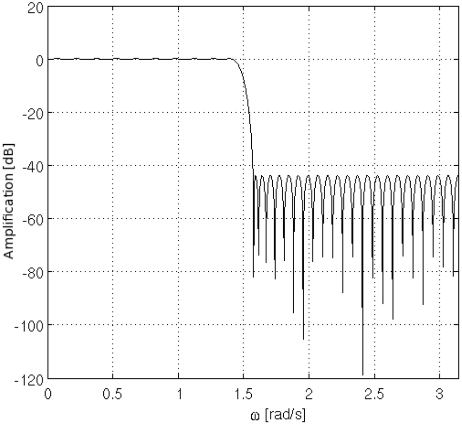

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Those results have been obtained in MATLAB with SDPT3 solver.

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Frequency response given by:

$$H^{\text{aim}}(\omega) = \begin{cases} (\frac{\omega}{0.9\pi} + \frac{1}{2})e^{-j60\omega} & \text{for } 0 \leqslant \omega \leqslant 0.45\pi, \\ 0 & \text{for } 0.5\pi \leqslant \omega \leqslant \pi. \end{cases}$$

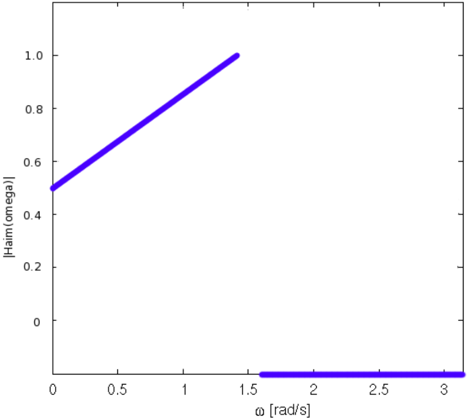

Figure: Absolute value of the desired transfer function

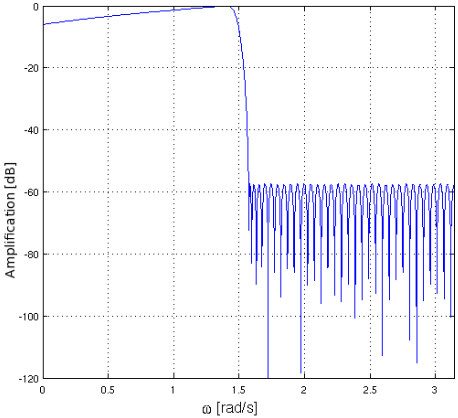

Figure: Absolute value of the transfer function $H(e^{j\omega})$ of the designed low-pass filter

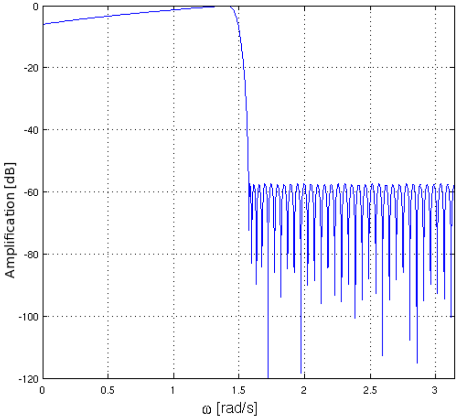

Figure: Absolute value of the transfer function $H(e^{j\omega})$ of the designed low-pass filter

It was caluclated with 300 sample frequencies (as earlier) and a filter length of $n = 120$.

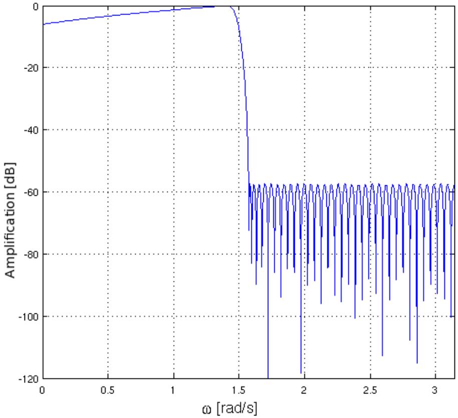

Figure: Absolute value of the transfer function $H(e^{j\omega})$ of the designed low-pass filter

It was caluclated with 300 sample frequencies (as earlier) and a filter length of $n = 120$.
It is actually a **6dB loss**, which is exactly what it should be for an absolute value of transfer function $\frac{1}{2}$ at $\omega = 0$.

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- IIR filter desgin with SOCP programming.
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References

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Thank you for your attention!